From the 4 run ~~of~~ in the 2^{3-1} experiment we get

$$l_{A_S} = \frac{y_2 + y_8}{2} - \frac{(y_3 + y_5)}{2} = \hat{A} + \widehat{BC}$$

In the same way

$$l_B = \hat{B} + \widehat{AC}$$

$$l_C = \hat{C} + \widehat{AB}$$

such that A is aliased with BC and B with AC and C with AB

$$l_I = \frac{1}{4} \sum_{ABC+} y_i = \frac{1}{8} \sum_{i=1}^{8} y_i + \frac{1}{8} \sum_{ABC+} y_i - \frac{1}{8} \sum_{ABC-} y_i$$

$$= \bar{y} + \frac{ABC}{2}$$

Generator and defining relation

The 2^{3-1} design can be generated by first constructing a 2^2 experiment in A and B and thereafter letting $C = AB$.

$C = AB$ is called the generator for the design.

$C = AB \iff CC = I = ABC$. I is called the defining relation. To find out which effects that are aliased (have the same signs) we can multiply by the defining relation.

$$AI = AABC = BC \Rightarrow A \equiv BC$$

$$BI = BABC = AC \Rightarrow B \equiv AC$$

$$CI = CABC = AB \Rightarrow C \equiv AB.$$

Constructing half fractions or 2^{p-1} designs

Construct a 2^{p-1} design in the factors $1, 2, \dots, p-1$ and let the design columns for the last factor be the $\pm\, 1 2 3 \dots (p-1)$ interaction column

Resolution in fractions of 2^p experiments.

Definition. A design is said to be of resolution R if no p-factor effect is aliased with an effect containing less than $R-p$ factors.

For a resolution R design we have

Main effects are aliased with $R-1$ factor interactions.

Two-factor interactions are aliased with $R-2$ factor interactions

Resolution III

Main effects are aliased with two-factor interactions

Example 2^{3-1}, $I = ABC$

Resolution IV

Main effects are aliased with three-factor interactions

Two-factor interactions are aliased with two-factor interactions

Example 2^{4-1}, $I = ABCD$

Resolution V

Main effects are aliased with four-factor interactions

Two-factor interactions with three-factor interactions.

Example 2^{5-1}, $I = ABCDE$.

Fractions of 2^p experiments

2^{5-2} i.e a quarter fraction of a 2^5 experiment.

Pick those eight runs for which ABD and ACE has a plus

This gives $I = ABD = ACE$

or Construct a 2^3 design in A, B and C. Then let $D = AB$ and $E = AC$.

We get. $I = ABD = ACE$ and $I^2 = ABD \cdot ACE = BCDE$

Defining relation $I = ABD = ACE = BCDE$ and $R = \underline{III}$.

Write $2^{5-2}_{\underline{III}}$. Hence $A \rightarrow A + BD + CE + ABCDE$

Resolution III designs

Designs with $2^p - 1$ factors in 2^p runs are called saturated. These are always resolution III designs.

Example Bicycle experiment

A = Seat	E = Raincoat
B = Dynamo	F = Breakfast
C = Handlebars	G = Tires
D = Gear	y = time to climb a hill

Construction

Construct a 2^3 design in A, B and C and then

$$\left.\begin{array}{ll} D = AB & F = BC \\ E = AC & G = ABC \end{array}\right\} \text{Generators}$$

$I = ABD = ACE = BCF = ABCG$

$I^2 = BCDE = ACDF = CDG = ABEF = BEG = AFG$

$I^3 = DEF = ADEG = BDFG = CEFG$

$I^4 = ABCDEFG$

Neglecting interactions of order three and higher.

$\hat{l}_A \rightarrow A + BD + CE + FG$

$\hat{l}_B \rightarrow B + AD + CF + EG$

$\hat{l}_C \rightarrow C + AE + BF + DG$

$\hat{l}_D \rightarrow D + AB + CG + EF$

$\hat{l}_E \rightarrow E + AC + BG + DF$

$\hat{l}_F \rightarrow F + BC + AG + DE$

$\hat{l}_G \rightarrow G + CD + BE + AF$.

There are 16 possible 2^{7-4} fractions according to the generator

$$\begin{array}{ll} D = \pm AB & F = \pm BC \\ E = \pm AC & G = \pm ABC \end{array}$$

For estimating main effects free of aliasing with two-factor interactions, run a new 2^{7-4} fraction where $\left.\begin{array}{ll} D = -AB & F = -BC \\ E = -AC & G = ABC \end{array}\right\}$ Genera